Exercises

Session 4 (section 4.6)

1 Measurement of impedance

Figure 1 shows a functional diagram of a device measuring a bioimpedance. The model of the impedance to measure in this example consists of a resistance R with a constant part R_0 and a varying part assumed sinusoidal at angular frequency ω_R and magnitude $R_1 < R_0$. The device is modelled as a current source driven by a sinewave with angular frequency $\omega_j \gg \omega_R$ and magnitude j_0.

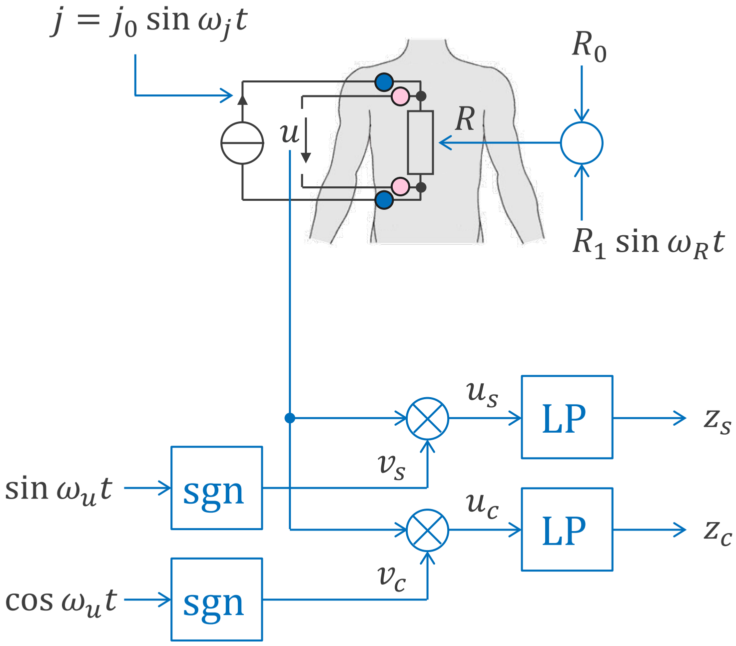

Figure 1: Functional diagram of a device measuring bioimpedance

1.1. Time and frequency graphs of signals

<u>Exercise statement</u>

Sketch the graphs of the resistance $R(t)$, current $j(t)$, and voltage $u(t)$ signals. Sketch their spectrum (graphs of $|R(\omega)|$, $|j(\omega)|$, $|u(\omega)|$).

1.2. Demodulation with square wave

The signal $u(t)$ results from an amplitude modulation of a sine carrier $j(t)$ and the signal $R(t)$. The amplitude modulation is a product, i.e., $u(t) = j(t)R(t)$. The operation consisting of recovering $R(t)$ from the signal $u(t)$ is called demodulation. It is not performed by the division of $u(t)$ by $j(t)$, first because $j(t)$ has zeros, but also because it is not practical to perform a division with electronic circuits. It is much easier to take advantage that the carrier is a sinewave, which means that multiplying again by $j(t)$ results in a $\sin^2 \omega_j t$ which is also equal to

$$\sin^2 \omega_j t = \frac{1}{2}\left(1 + \sin 2\omega_j t\right)$$

Therefore, the signal $u(t)$ multiplied by $\sin \omega_j t$ is:

$$u(t) \sin \omega_j t = \frac{j_0}{2}\left(1 + \sin 2\omega_j t\right) R(t)$$

It is then easy to use a low-pass filter LP to remove the $\sin 2\omega_j t$ component and obtain:

$$\mathrm{LP} * \left(u(t) \sin \omega_j t\right) = \frac{j_0}{2} R(t)$$

Multiplying by a square wave is easier than multiplying by a sinewave, since the electronic circuit of such multiplication simply consists of changing the sign of the signal every other half period. However, a ± 1 square wave has odd harmonics ($k = \cdots, -3, -1, 1, 3, \cdots$) with magnitude:

$$\frac{2}{k\pi}$$

<u>Exercise statement</u>

The diagram in Figure 1 demodulates with a square wave. Sketch the spectrum of the demodulated signals u_s and u_c (see Figure 1).